L.42

# PRE-LEAVING CERTIFICATE EXAMINATION, 2012

# APPLIED MATHEMATICS – HIGHER LEVEL

## TIME : 2½ HOURS

Six questions to be answered.  All questions carry equal marks.

A *Formulae and Tables* booklet may be used during the examination.

Take the value of $g$ to be 9.8 m $s^{-2}$.

**Marks may be lost if necessary work is not clearly shown.**

**1.** **(a)** A particle falls from rest from a point $X$ under gravity. After it has fallen a distance $a$ metres, another particle is given a downward speed of $\sqrt{8ga}$ m $s^{-1}$ from the same starting point $X$.

**(i)** How long is the first particle moving before the second particle starts moving?

**(ii)** If the particles collide $t$ seconds after the second particle starts moving, express $t$ in terms of $a$ and $g$.

**(iii)** Find, in terms of $a$, the distance from $X$ to the point where they collide.

**(b)** Two trains, S and T, travel on parallel rails in the same direction with uniform accelerations of $f$ and $\frac{3}{2}f$, respectively.
At the same instant, both trains pass a signal box $P$ with speeds $u$ and $\frac{1}{2}u$, respectively.
The trains are once again level when they pass the next signal box $Q$.

**(i)** Show that the greatest distance that S is ahead of T is $\frac{u^2}{4f}$.

**(ii)** Find, in terms of $u$, the speed of each train as it passes $Q$.

**2.** **(a)** A battleship, whose top speed is 24 km/h, sets out to intercept a convoy which is sailing due east at 16 km/h. The convoy is 29 km away in the direction 30° south of east, and it maintains its course.

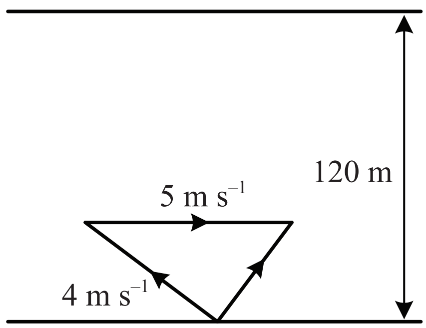
Find **(i)** the shortest time in which the battleship can reach the convoy

**(ii)** the direction in which the battleship should travel, correct to the nearest degree.

**(b)** A man can swim at 4 m $s^{-1}$ in still water. He swims across a river of width 120 metres. The river flows with a constant speed of 5 m $s^{-1}$ parallel to the straight banks.
He wishes to cross by the shortest path.

Find **(i)** the direction he should take

**(ii)** the time taken to cross the river by the shortest path.



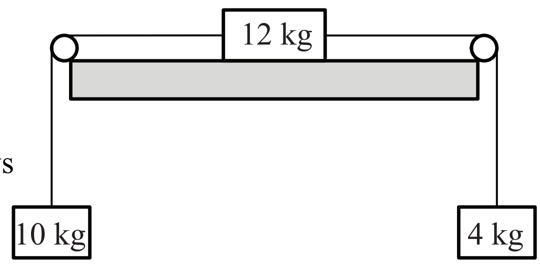
**3.** **(a)** A particle is projected from a point on a horizontal plane so that it just clears a vertical wall of height $\frac{a}{2}$ metres at a horizontal distance of $a$ from the point of projection.
The particle strikes the plane at a horizontal distance of $3a$ metres beyond the wall.

Show that the angle of projection, measured to the horizontal, is $\tan^{-1}\frac{2}{3}$.

**(b)** A particle is projected up a plane which is inclined at an angle $\beta$ to the horizontal. The angle of projection, measured to the inclined plane, is $\alpha$.
The plane of projection is vertical and contains the line of greatest slope.
The particle strikes the plane after $t$ seconds.

Show that the component of the velocity parallel to the inclined plane after $t$ seconds is negative if $2\tan\alpha\tan\beta > 1$.

**4.** **(a)** A particle of mass 12 kg rests on a rough horizontal table. It is attached by two horizontal inelastic strings to particles of masses of 10 kg and 4 kg which hang freely over smooth pulleys at opposite edges of the table.

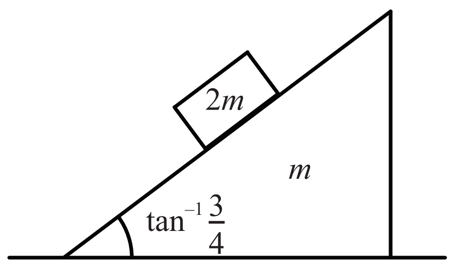



The coefficient of friction between the 12 kg mass and the table is $\mu$.

**(i)** Show that if $\mu = \frac{1}{4}$, the common acceleration of the particles is $\frac{3g}{26}$.

**(ii)** Find the least value of $\mu$ for which the particles will not move.

**(b)** A smooth particle of mass $2m$ rests on the smooth face of a wedge of mass $m$ and angle $\tan^{-1}\frac{3}{4}$ The wedge is free to move on a rough horizontal table, the coefficient of friction being $\frac{1}{4}$.
The system is released from rest.




**(i)** Show, on separate diagrams, the forces acting on the wedge and the particle.

**(ii)** Show that the acceleration of the wedge is $\frac{39g}{148}$.

5. **(a)** A smooth sphere P, of mass $3m$ and moving with speed $4u$, collides directly with another smooth sphere Q, of mass $2m$ and moving in the opposite direction with a speed $3u$. Sphere P is brought to rest by the collision.

**(i)** Show that the velocity of Q is reversed in direction but unchanged in magnitude by the collision.

**(ii)** Find $e$, the coefficient of restitution.

**(iii)** Determine the fraction of the total kinetic energy that was lost during the impact.

**(b)** Two smooth spheres, A and B, of equal radii but of masses 20 kg and 10 kg, respectively, lie on a smooth horizontal table. B is at rest and A, moving at 4 m $s^{-1}$, strikes B obliquely, the direction of A at the moment of impact making an angle of 60° with the line of centres.

The coefficient of restitution between the spheres is $\frac{1}{2}$.

Find **(i)** the speed of A and the speed of B directly after impact

**(ii)** the angle between their directions of motion after impact

**(iii)** the loss in kinetic energy due to the impact.

6. **(a)** A particle moves with simple harmonic motion and performs two complete oscillations per second. Its speed when it is $\frac{\sqrt{3}}{4}$ metres from the centre of the motion is half the maximum speed.

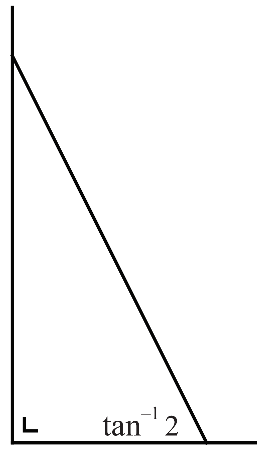
**(i)** Find the amplitude of the motion.

**(ii)** The speed of the particle is three quarters of the maximum speed at two points. Find the distance between these two points.

**(b)** Two light, inextensible strings AB and BC, each of length $l$, are attached to a particle of mass $m$ at B. The other ends, A and C, are fixed to two points in a vertical line such that A is distant $l$ above C. The particle describes a horizontal circle with constant angular velocity.

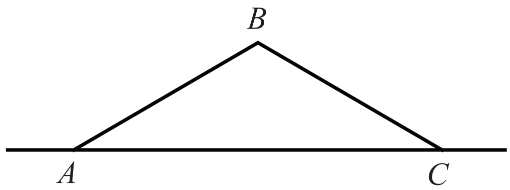
Find the greatest periodic time for which both strings will be taut.

**7.** **(a)** A uniform ladder, of weight $W$ and length $2l$, rests with one end on rough horizontal ground and the other end against a smooth vertical wall. The ladder makes an angle of $\tan^{-1} 2$ with the ground and is in a vertical plane perpendicular to the wall.
The coefficient of friction between the ladder and the ground is $\frac{1}{3}$.




Find the maximum distance up the ladder that a boy of weight $2W$ can climb before the ladder begins to slip.

**(b)** Two equal uniform rods $AB$ and $BC$, of weights $W$ and $2W$, respectively, are freely jointed at $B$. They rest in a vertical plane with $A$ and $C$ on rough horizontal ground. The coefficient of friction at both $A$ and $C$ is $\mu$. When the angle $ABC$ is 120° slipping is about to occur.

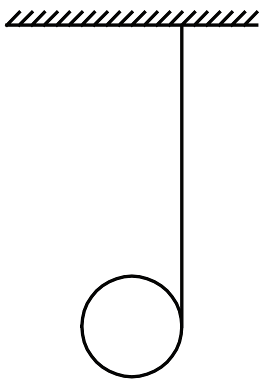



**(i)** Explain why slipping occurs at $A$ before $C$.

**(ii)** Find the value of $\mu$.

**8.** **(a)** Prove that the moment of inertia of a uniform circular disc of mass $m$ and radius $r$ about an axis through its centre perpendicular to its plane is $\frac{1}{2}mr^2$.

**(b)** A light string is wound around the rim of a uniform disc of radius $2r$ and mass $2m$.
One end of the string is attached to the rim of the disc and the other end is attached to a fixed point above the disc, with the plane of the disc vertical.
When the disc is released from rest, it falls vertically and the string unwinds.

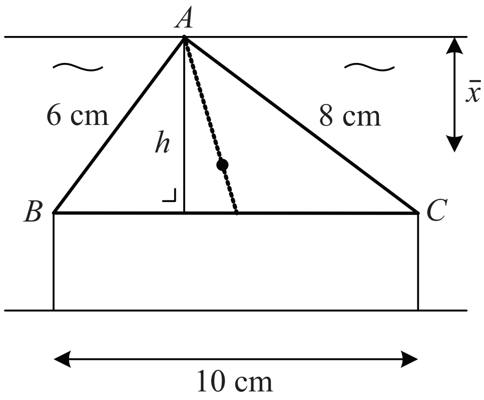


Find **(i)** the vertical acceleration of the disc

**(ii)** the tension in the string

**(iii)** the time taken for the disc to fall a distance $r$.

**9.** **(a)** The triangle $ABC$ is right angled at $A$, with $|AB| = 6$ cm, $|AC| = 8$ cm and $|BC| = 10$ cm. This triangular lamina is held submerged in water with $A$ at the surface of the water and $[BC]$ horizontal. $h$ is the height of the triangle, as shown. $\bar{x}$ is the depth of the centre of mass of the triangle below the water surface.




**(i)** Show that $\bar{x} = \frac{2}{3}h$ and hence find $\bar{x}$.

**(ii)** Find the thrust on the triangular lamina.

**(b)** A compound object is made of some plastic of relative density 2.5 and of some metal of relative density 3.
The compound object weighs 12.544 N in water and 9.7216 N in a liquid of relative density 1.4.

Find the volume of the plastic and the volume of the metal in the object.

**10.** **(a)** Solve the differential equation

$$x\frac{dy}{dx} = y(3 - x)$$

given that $y = \frac{2}{e}$ when $x = 1$.

**(b)** A particle starts with a speed of $15 \text{ m s}^{-1}$ and moves in a straight line.
The particle is subjected to a force which produces an acceleration which is initially $0.5 \text{ m s}^{-2}$ and which increases uniformly with the distance moved, having a value of $1 \text{ m s}^{-2}$ when the particle has moved a distance of 10 metres.

If $v \text{ m s}^{-1}$ is the speed of the particle when it has moved a distance of $x$ metres,

**(i)** prove that while the particle is in motion,
$$v\frac{dv}{dx} = \frac{x + 10}{20}$$

**(ii)** calculate the distance moved by the particle while its speed increases to $5\sqrt{33} \text{ m s}^{-1}$.

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