L.42

PRE-LEAVING CERTIFICATE EXAMINATION, 2013

APPLIED MATHEMATICS – HIGHER LEVEL

TIME : 2½ HOURS

Six questions to be answered. All questions carry equal marks.
A *Formulae and Tables* booklet may be used during the examination.
Take the value of g to be 9.8 m s^{-2}.
Marks may be lost if necessary work is not clearly shown.

1. **(a)** A particle moving in a straight line with constant acceleration passes three points K, L and M, and has speeds u and $u\sqrt{71}$ at K and M, respectively.

(i) If L is the midpoint of $[KM]$, express the speed of the particle at L in terms of u.

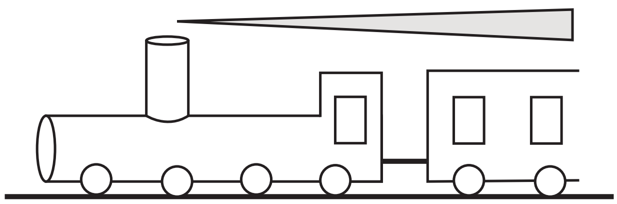
(ii) If the time taken by the particle to travel from L to M is 7 seconds, express the time taken to travel from K to L in the form $\sqrt{a}+b$ seconds, where $a, b \in$ □ .

(b) Two particles, A and B, travelling in the same direction, pass the point P at the same instant. A is moving with velocity u_1 and retardation f_1 and B with velocity $u_2 (< u_1)$ and retardation $f_2 (< f_1)$.

(i) Show that B will overtake A after a time $\dfrac{2(u_1 - u_2)}{f_1 - f_2}$.

(ii) Find, in terms of u_1, u_2, f_1 and f_2, the greatest lead that A has over B during this motion.

2. **(a)** An old train, drawn by a steam engine, is moving due north at 20 km h^{-1}. The wind is blowing at 10 km h^{-1} from the southeast.



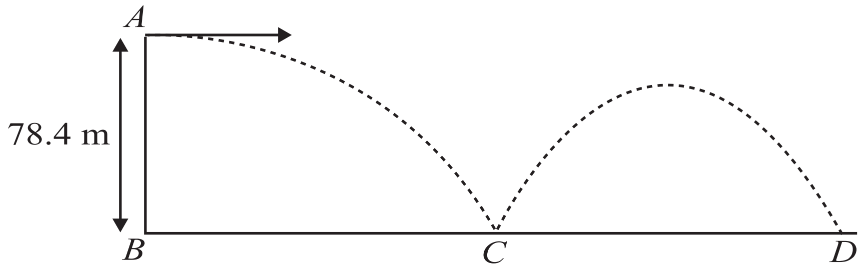
As the smoke leaves the engine, it immediately takes the velocity of the wind.

Find the angle, in degrees, that the smoke trail makes with the engine correct to two places of decimals.

(b) Relative to a ship which is travelling due north at a speed of 10 km h^{-1}, the velocity of a speedboat is in the direction North 45° East. Relative to a second ship which is travelling due south at 10 km h^{-1}, the velocity of the speedboat is in the direction North 30° East.

Find the true speed and direction of the speedboat.

3. **(a)** A particle is projected horizontally from a point A, which is 78.4 m vertically above a point B on horizontal ground. Its initial speed is $2g$ m s^{-1}. It first strikes the horizontal plane at C, and next strikes the horizontal plane at D.



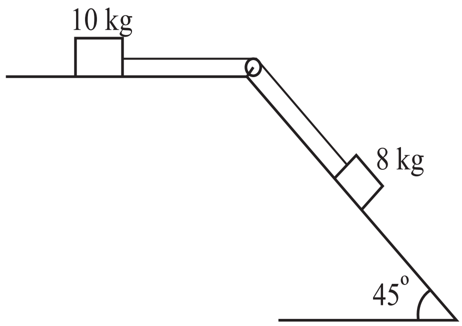
If $|BC| = |CD|$ find the coefficient of restitution, e.

(b) A particle is projected from a point P, up a plane inclined at an angle $\tan^{-1}\frac{1}{6}$ to the horizontal. The direction of projection makes an angle α with the inclined plane. (The plane of projection is vertical and contains the line of greatest slope.)

If the particle strikes the inclined plane while travelling horizontally, show that $\tan\alpha = \frac{3}{19}$.

4. **(a)** A particle of mass 10 kg is connected by a light inextensible string passing over a light smooth pulley at the edge to another particle of mass 8 kg on a plane inclined at 45° to the horizontal.

The coefficient of friction between the particles and the planes is $\frac{1}{4}$.



Find **(i)** the acceleration of the particles

(ii) the tension in the string, correct to two places of decimals.

(b) A wedge of mass 10 kg sits on a smooth horizontal plane. A particle of mass 8 kg sits on a face of the wedge inclined at an angle of 45° to the horizontal. The coefficient of friction between the particle and the wedge is $\frac{1}{\sqrt{2}}$.

The system is released from rest.

Find **(i)** the acceleration of the wedge

(ii) the acceleration of the particle relative to the wedge.

5. **(a)** A smooth sphere A of mass m collides directly with a smooth sphere B of mass $2m$ which is at rest. During the impact, $\frac{10}{27}$ of the original kinetic energy is lost.

Find the coefficient of restitution between the spheres.

(b) A smooth sphere A collides with an equal smooth sphere B which is at rest. The directions of motion of A before and after impact make angles α and β, respectively with the line of centres.

(i) Express β in terms of α and e, the coefficient of restitution.

(ii) If $\tan\alpha = \frac{1}{2}$ and $e = \frac{1}{2}$ show that A is deflected through the angle $\tan^{-1} = \frac{3}{4}$ by the collision.

6. **(a)** The displacement, x metres, of a moving particle at any time t seconds is given by the equation

$$x = 8\sin 3t + 6\cos 3t\,.$$

(i) Show that the motion is simple harmonic motion.

(ii) By considering the velocity of the particle, calculate the amplitude of the motion.

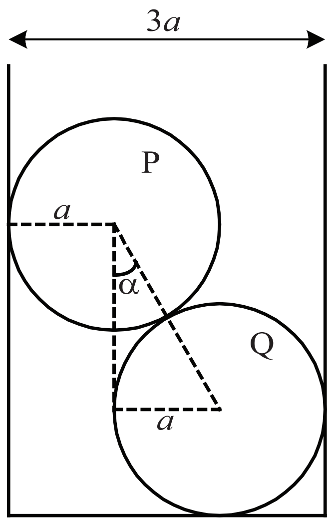
(iii) Find the first time that the particle is at the centre of the motion.

(b) A small ring of mass m can slide on a smooth circular wire of radius r and centre O which is fixed in a vertical plane. From a point on the wire at a vertical distance $\frac{r}{2}$ above O, the ring is given a velocity $\sqrt{gr}$ along the downward tangent to the wire.

(i) Show that the particle will just reach the highest point of the wire.

(ii) Find the reaction between the ring and the wire when the ring is at the same horizontal level as O.

7. **(a)** The diagram shows two smooth vertical walls at a distance $3a$ apart, with two smooth cylinders, P and Q, each of weight W and radius a. The axes of the cylinders are horizontal and parallel to the walls. Q rests on smooth horizontal ground. α is the angle between the line of centres and the vertical.



(i) Find α.

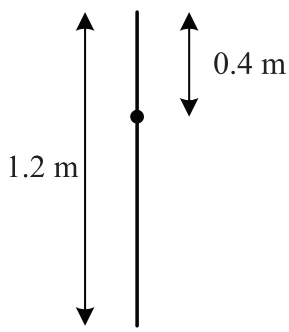
(ii) Find, in terms of W, the magnitude of the reaction between the cylinders.

(iii) Find, in terms of W, the magnitude of the reaction between each cylinder and the corresponding wall.

(b) A thin uniform rod rests with one end on a smooth plane inclined at 60° to the horizontal and the other end on rough horizontal ground. If the rod is about to slip when inclined at 30° to the horizontal, show that the coefficient of friction between the rod and the ground is $\frac{1}{\sqrt{3}}$.

8. **(a)** Prove that the moment of inertia of a uniform rod of mass m and length $2l$ about an axis through its centre perpendicular to the rod is $\frac{1}{3}ml^2$.

(b) A uniform rod of mass m and of length 1.2 m swings in a vertical plane about a horizontal axis through the rod at a distance of 0.4 m from its upper end.



(i) If v m s^{-1} is the velocity of the lower end when the rod is vertical, prove that the rod will make a complete revolution if $v \geq 5.6$ m s^{-1}.

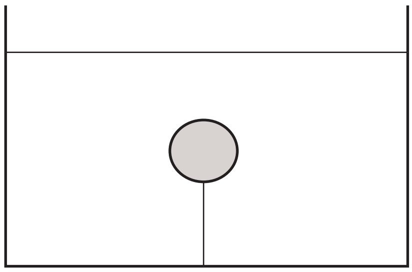
(ii) If a mass m is attached to each end of the rod and the compound body is released from rest in a horizontal position, find the angular speed when the rod is next vertical.

9. **(a)** Mercury, of relative density 13.6, is poured into a vertical U-tube. Alcohol, of relative density 0.8, is then poured into one limb of the U-tube until it occupies a height of 6.8 cm.

(i) What is the difference in the free surface levels of the mercury and the alcohol?

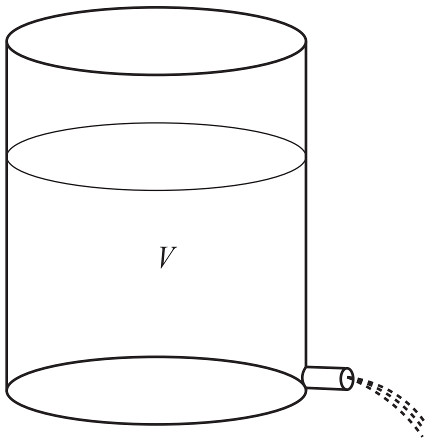
(ii) If chloroform, of relative density 1.5, is then poured into the other limb until the free surfaces of the chloroform and the alcohol are at the same level, what is the height of the chloroform in the tube?

(b) A uniform sphere is held completely immersed in water by means of a string tied to a point on the base of the container. The tension in the string is 2.94 N. When the water is replaced by a liquid of relative density 1.2, the tension in the string is 5.88 N.



Find the mass of the sphere.

10. **(a)** The diagram shows water flowing out of a container through a tap. It can be shown that the volume V litres after a time t minutes is given by $\frac{dV}{dt} = -k\sqrt{V}$.
Initially, the container holds 100 litres of water. After 2 minutes, the volume of water is 96 litres. Find



(i) the value of k, correct to three places of decimals

(ii) the volume of water left in the container after 10 minutes.

(b) A car of mass m moving on a straight horizontal road is subject to a constant resistance R. The engine is working at a constant rate kR.

(i) Given that in time t the car accelerates from rest to a speed v, where $v < k$ find an expression for t in terms of m, R, k and v.

(ii) Find the time taken by the car to increase its speed from $\frac{1}{3}k$ to $\frac{2}{3}k$.

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