PRE-LEAVING CERTIFICATE EXAMINATION, 2014

APPLIED MATHEMATICS – HIGHER LEVEL

TIME: 2½ HOURS

Six questions to be answered. All questions carry equal marks.

A *Formulae and Tables* booklet may be used during the examination.

Take the value of g to be 9.8 m s^{-2}.

Marks may be lost if necessary work is not clearly shown.

1. **(a)** An underground train has to travel a distance d from rest at one station to rest at the next station. The train has a maximum acceleration of a and a maximum deceleration of b.

If the train makes the journey in the shortest possible time without having to travel at constant speed, show that the speed limit on the track, v, satisfies

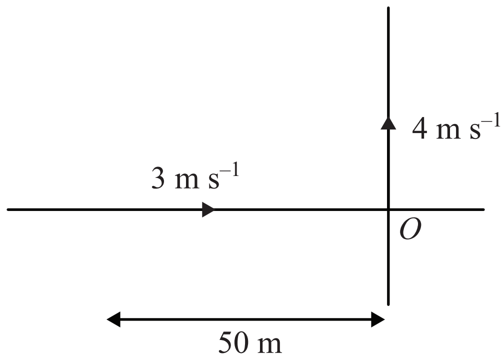
$$v \geq \sqrt{\frac{2abd}{a+b}}.$$

(b) A stone is dropped from the top of a tower which is located on horizontal ground through the base of the tower. One second later, another stone is thrown vertically downwards from the same point with a speed of 15 m s^{-1}.

(i) If the two stones reach the ground simultaneously, find the height of the tower.

(ii) If a third stone is thrown downwards from the same point half a second after the second stone, what initial speed must it have to reach the ground at the same time as the first two stones?

2. **(a)** Two straight roads cross at right angles at O. On one road, a car is travelling due north at 4 m s^{-1}. As the car passes through O, a bus is 50 m from O and travelling east towards O at a speed of 3 m s^{-1}.



(i) Find the velocity of the bus relative to the car.

(ii) Calculate the shortest distance between the car and the bus and the time at which this occurs.

(iii) Find the length of time during which the car and the bus are within 41 m of one another.

(b) A ship P is travelling in the direction 38° north of east. To an observer on another ship Q, which is travelling 18° south of east at 33 km h^{-1}, P appears to be travelling in the direction 67° north of west.

(i) Find the actual speed of P, correct to one decimal place.

(ii) Find the magnitude of the velocity of P relative to Q, correct to one decimal place.

(iii) Q reduces its speed, without changing direction, so that P appears to be travelling due north. Find the reduced speed of Q, correct to one decimal place.

3. **(a)** A particle is projected from a point on a horizontal plane so that it just clears a vertical wall of height $\frac{a}{2}$ at a horizontal distance of a from the point of projection and strikes the plane at a horizontal distance $3a$ **beyond the wall**.

(i) Express the range of the particle on the horizontal plane in terms of a.

(ii) If the angle of projection measured to the horizontal is $\tan^{-1}\frac{2}{k}$, find the value of k.

(b) A plane is inclined at an angle β to the horizontal.

A body is projected up the plane with velocity u m s^{-1} at an angle of 30° to the inclined plane.

The plane of projection is vertical and contains the line of greatest slope. The particle strikes the plane at right angles.

(i) Find the value of c and the value of d if $\tan\beta = \frac{\sqrt{c}}{d}$.

(ii) If the range of the body on the inclined plane is $10\sqrt{21}$ m, find the value of u.

4. **(a)** A light inextensible string is attached at one end to a particle A, of mass 5 kg, hanging freely.

The string passes over a smooth fixed pulley and its other end is attached to another particle B, of mass 3 kg, which is held at a height of 5 m above the horizontal plane.

Initially the string is slack. The system is released from rest.

After B has fallen through 1 m, the string becomes taut. Find

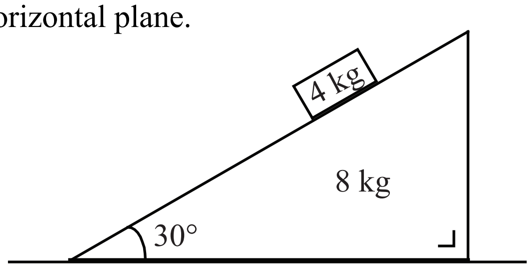
(i) the speed of B directly after the string becomes taut

(ii) the nearest that B gets to the horizontal plane.

(b) A wedge of mass 8 kg sits on a smooth horizontal plane. A particle of mass 4 kg sits on a face of the wedge which is inclined at 30° to the horizontal. The coefficient of friction between the particle and the wedge is $\frac{1}{2}$.

The system is released from rest.

Find the acceleration of the wedge.



5. **(a)** Two smooth spheres, P and Q, of equal mass, are travelling directly towards each other with speeds u and $2u$, respectively.

(i) Determine if it is possible for the sphere Q to be at rest after the collision.

(ii) If the fraction of the kinetic energy lost during the collision is $\frac{27}{40}$, find the value of e, the coefficient of restitution.

(b) A smooth sphere A, of mass m, collides obliquely with a smooth sphere B, of mass $2m$, which is at rest.

Before the collision, A has a velocity of u in a direction which makes an angle of 30° with the line of centres.

If A is deflected through an angle of 90° by the collision, find the value of e, the coefficient of restitution.

6. **(a)** A particle is performing simple harmonic motion of amplitude $0 \cdot 8$ m about a fixed point O.

A and B are two points on the path of the particle such that $|OA| = 0 \cdot 6$ m and $|OB| = 0 \cdot 4$ m. The particle takes 2 seconds to travel from A to B.

Find, correct to two decimal places, the periodic time of the motion if

(i) A and B are on the same side of O

(ii) A and B are on opposite sides of O.

(b) A particle P is moving on the inner surface of a smooth hemispherical bowl with centre O and radius $2a$.

The particle is describing a horizontal circle, centre C, with angular speed $\sqrt{\frac{g}{a}}$.

Find **(i)** the magnitude of the force exerted on P by the surface of the bowl

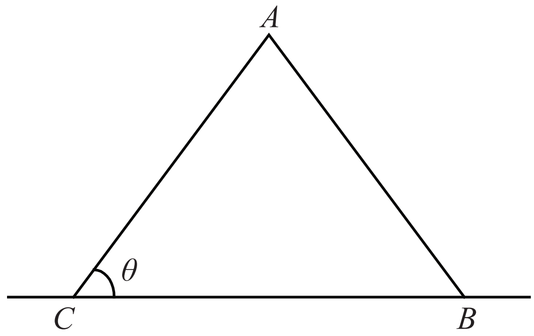
(ii) the depth of C below O.

7. Two uniform rods, AB and AC, each of length $2a$ and weight W, are smoothly jointed at A. The end C is freely hinged to a point on a rough horizontal plane.

The end B rests on the plane and is on the point of slipping.

Both rods are in a vertical plane.

The coefficient of friction between B and the plane is μ and the angle between AC and the horizontal is θ.



(i) Show, on separate diagrams, all the forces acting on the structure ABC and on the rod AB.

(ii) Prove that $\mu = \dfrac{1}{2\tan\theta}$.

(iii) Find, in terms of W and μ, the magnitude and direction of the reaction force at the end B resting on the plane.

(iv) Find, in terms of W and θ, the magnitude of the reaction at the joint A.

8. (a) Prove that the moment of inertia of a uniform rod of mass m and length $2l$ about an axis through its centre perpendicular to the rod is $\frac{1}{3}ml^2$.

(b) A uniform rod AB of mass m and length $2l$ is free to rotate in a vertical plane about a horizontal axis through A. A particle of mass $2m$ is attached to the rod at B.

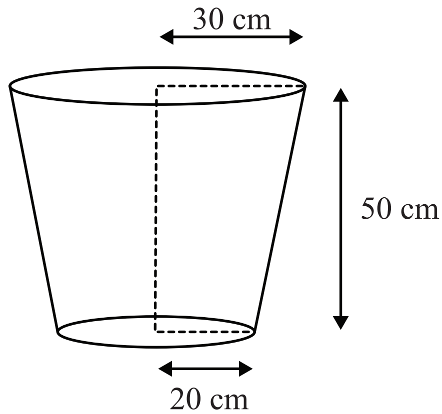
The system is released from rest with B vertically above A.

(i) Find the angular velocity when the system is next vertical.

At this point the mass at B falls off.

(ii) Find the height of B when it next comes to rest.

9. **(a)** A bucket, in the shape of a frustum, has a height of 50 cm, a base of radius 20 cm and a top of radius 30 cm.

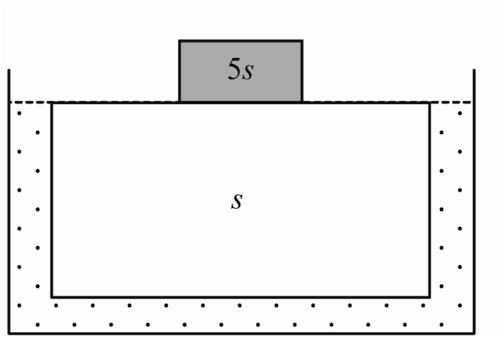


Find, in terms of π and g,

(i) the thrust on the base if the bucket is filled with water

(ii) the thrust on the curved side if water is poured into the bucket to a depth of 25 cm.

(b) A block of wood, of volume V and relative density s, floats in water.



A smaller block of metal, of volume V_1 and relative density $5s$, is placed on top of the wood, such that the upper surface of the wooden block is in the free surface of the water.

Prove that $\dfrac{V}{V_1} = \dfrac{5s}{1-s}$.

10. **(a)** Solve the differential equation

$$x\frac{dy}{dx} = y(1+x)$$

given that $y = 3$ when $x = 1$.

(b) A particle of mass m moves with a velocity v m s^{-1} in a straight line through a medium in which the resistance to motion is mkv^3, where k is a constant. No other force acts on the body.

The velocity of the particle falls from 15 m s^{-1} to $7 \cdot 5$ m s^{-1} in a time of t_1 seconds.

Show that the distance travelled in this time is $10t_1$ m.

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