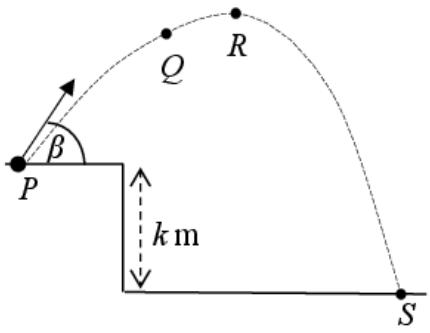
Question 1

3. A particle is projected from a point P, as shown in the diagram, with an initial speed of 74 m s^{-1} at an angle β to the horizontal, where $\tan\beta = \frac{35}{12}$.

The particle reaches point Q after 4 seconds of motion.
R is the highest point reached by the particle.



Find **(i)** the initial velocity of the particle in terms of $\vec{i}$ and $\vec{j}$

(ii) the velocity of the particle at point Q in terms of $\vec{i}$ and $\vec{j}$

(iii) the displacement of R from P in terms of $\vec{i}$ and $\vec{j}$

(iv) the value of k, given that the particle reaches S after 16 seconds of motion.

(i) $\vec{u} = 74\cos\beta\,\vec{i} + 74\sin\beta\,\vec{j}$

$= 24\,\vec{i} + 70\,\vec{j}$ — 10

(ii) $\vec{v}_Q = 24\,\vec{i} + \{70 - 10\times 4\}\vec{j}$

$= 24\,\vec{i} + 30\,\vec{j}$ — 10

(iii) $\vec{r}_R = 24t\,\vec{i} + \{70t - \frac{1}{2}gt^2\}\vec{j}$ — 5

$= 24(7)\,\vec{i} + \{70(7) - 5(7)^2\}\vec{j}$ — 5

$= 168\,\vec{i} + 245\,\vec{j}$ — 5

(iv) $-k = 70(16) - 5(16)^2$ — 5

$-k = 1120 - 1280$ — 5

$k = 160 \text{ m}$ — 5

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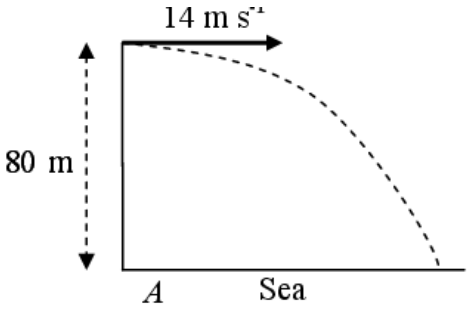
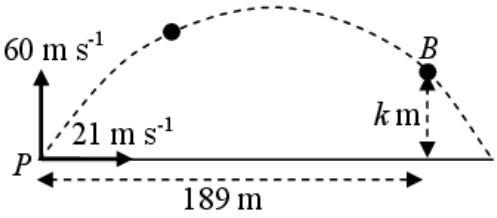
Question 2

3. **(a)** A particle is projected horizontally with an initial speed of 14 m s^{-1} from the top of a straight vertical cliff of height 80 m.

How far from the foot of the cliff will it hit the sea?



(b) A particle is projected with initial velocity $21\,\vec{i} + 60\,\vec{j}$ m s^{-1} from point P on a horizontal plane.
A and B are two points on the trajectory (path) of the particle.
The particle reaches point A after 4 seconds of motion.
The displacement of point B from P is $189\,\vec{i} + k\,\vec{j}$ m.



Find **(i)** the velocity of the particle at A in terms of $\vec{i}$ and $\vec{j}$
(ii) the speed and direction of the particle at A
(iii) the value of k.

(a)	$80 = 0 + 5 \times t^2$	
	$t = 4$ s	10
	$d = 14 \times 4 = 56$ m	10
(b)(i)	$v_y = u + at$	
	$= 60 - 10 \times 4 = 20$	5
	$\vec{v} = 21\,\vec{i} + 20\,\vec{j}$	5
(ii)	$\lvert\vec{v}\rvert = \sqrt{21^2 + 20^2}$	
	$= 29$ m s^{-1}	5
	$\beta = \tan^{-1}\frac{20}{21} = 43.6°$	5
(iii)	$21t = 189$	
	$t = 9$	5
	$s_y = ut + \frac{1}{2}at^2$	
	$k = 60 \times 9 - 5 \times 9^2$	
	$= 135$	5

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3. A particle is projected from a point on horizontal ground with an initial speed of 82 m s^{-1} at an angle β to the horizontal, where $\tan\beta = \frac{40}{9}$.

Find **(i)** the initial velocity of the particle in terms of $\vec{i}$ and $\vec{j}$
(ii) the time taken to reach the maximum height
(iii) the maximum height of the particle above ground level
(iv) the range
(v) the two times at which the height of the particle is 275 m.

(i) $\vec{V} = 82\cos\beta\,\vec{i} + 82\sin\beta\,\vec{j}$

$= 18\,\vec{i} + 80\,\vec{j}$ 10

(ii) $v_y = u + at$

$0 = 80 - 10t$

$t = 8 \text{ s}$ 10

(iii) $s_y = ut + \frac{1}{2}at^2$

$= 80 \times 8 - 5 \times 64$

$= 320 \text{ m}$ 10

(iv) $|AB| = 18 \times 16$

$= 288 \text{ m}$ 10

(v) $s_y = ut + \frac{1}{2}at^2$

$275 = 80 \times t - 5 \times t^2$

$t^2 - 16t + 55 = 0$

$t = 5, \quad t = 11 \text{ s}$ 10

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3. **(a)** A ball is kicked from a point P on horizontal ground with a speed of $20\ \text{m s}^{-1}$ at $45°$ to the horizontal.

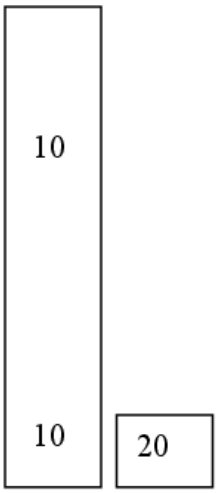
The ball strikes the ground at Q.

Find **(i)** the time it takes the ball to travel from P to Q

(ii) $|PQ|$, the distance from P to Q.

(i)
$$s_y = ut + \tfrac{1}{2}at^2$$
$$0 = 20\sin 45 \times t - 5t^2$$
$$t = 2\sqrt{2}\ \text{s}$$

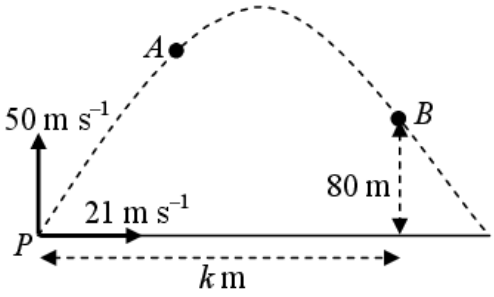
(ii)
$$s_x = ut + \tfrac{1}{2}at^2$$
$$|PQ| = 20\cos 45 \times t + 0$$
$$= 20 \times \frac{1}{\sqrt{2}} \times 2\sqrt{2}$$
$$= 40\ \text{m}$$



3 **(b)** A particle is projected with initial velocity $21\,\vec{i} + 50\,\vec{j}$ m s^{-1} from point P on a horizontal plane.

A and B are two points on the trajectory (path) of the particle.

The particle reaches point A after 3 seconds of motion.



The displacement of point B from P is $k\,\vec{i} + 80\,\vec{j}$ metres.

Find **(i)** the velocity of the particle at A in terms of $\vec{i}$ and $\vec{j}$
(ii) the speed and direction of the particle at A
(iii) the value of k.

(i)
$$v = u + at$$
$$v_x = 21 + 0$$
$$= 21$$ 5

$$v_y = 50 - 10 \times 3$$
$$= 20$$

$$v = 21\,\vec{i} + 20\,\vec{j}$$ 5

(ii)
$$|v| = \sqrt{21^2 + 20^2}$$
$$= 29 \text{ m s}^{-1}$$ 5

$$\alpha = \tan^{-1}\left(\frac{20}{21}\right)$$
$$= 43{\cdot}6°$$ 5

(iii)
$$80 = 50t - 5t^2$$
$$t^2 - 10t + 16 = 0$$
$$(t-2)(t-8) = 0$$
$$t = 8$$ 5

$$s_x = ut + \tfrac{1}{2}at^2$$
$$k = 21 \times 8$$
$$= 168$$ 5

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