## Newton's Law and Connected Particles (Dynamics)

4. (a) A particle of mass 3 kg is connected to a particle of mass 5 kg by a taut, light, inextensible string which passes over a smooth light pulley at the edge of a rough horizontal table.

The coefficient of friction between the 3 kg mass and the table is $\frac{2}{3}$.


The system is released from rest.
(i) Show on separate diagrams the forces acting on each particle.
(ii) Find the common acceleration of the particles.
(iii) Find the tension in the string.
(b) Masses of 8 kg and 2 kg are connected by a light inelastic string which passes over a smooth light pulley as shown in the diagram.

The 8 kg mass lies on a smooth plane which is inclined at $30^{\circ}$ to the horizontal.

The 2 kg mass hangs vertically.
The system is released from rest.

(i) Find the common acceleration of the masses.
(ii) Find the tension in the string.
4. (a) Two particles of masses 5 kg and 7 kg are connected by a taut, light, inextensible string which passes over a smooth light pulley.

The system is released from rest.
Find (i) the common acceleration of the particles
(ii) the tension in the string.

(b) Masses of 8 kg and 10 kg are connected by a taut, light, inextensible string which passes over a smooth light pulley as shown in the diagram.

The 8 kg mass lies on a rough horizontal plane and the coefficient of friction between the 8 kg mass and the plane is $\frac{1}{2}$.

The 10 kg mass lies on a
 smooth plane which is inclined at $30^{\circ}$ to the horizontal.

The system is released from rest.
(i) Show on separate diagrams the forces acting on each particle.
(ii) Find the common acceleration of the masses.
(iii) Find the tension in the string.
4. (a) Two particles of masses 3 kg and 2 kg are connected by a taut, light, inextensible string which passes over a smooth light pulley at the edge of a smooth horizontal table. The system is released from rest.
(i) Show on separate diagrams the forces acting on each particle.
(ii) Find the common acceleration of the particles.

(iii) Find the tension in the string.
(b) A particle of mass 2 kg is released from rest and slides down a rough plane which is inclined at an angle $\alpha^{0}$ to the horizontal, where $\tan \alpha=\frac{4}{3}$.
The coefficient of friction between the particle and the plane is $\frac{1}{2}$.

(i) Show on a diagram the forces acting on the particle.
(ii) Find the acceleration of the particle.
4. (a) Two particles of masses 9 kg and 5 kg are connected by a taut, light, inextensible string which passes over a smooth light pulley.

The system is released from rest.
Find (i) the common acceleration of the particles
(ii) the tension in the string.
(b) Masses of 3 kg and 6 kg are
 connected by a taut, light, inextensible string which passes over a smooth light pulley as shown in the diagram.

The 3 kg mass lies on a rough horizontal plane and the coefficient of friction between the 3 kg mass and the plane is $\mu$.


The 6 kg mass lies on a smooth plane which is inclined at $30^{\circ}$ to the horizontal.

When the system is released from rest each mass travels
1 metre in $\sqrt{2}$ seconds.
Find (i) the common acceleration of the masses
(ii) the tension in the string
(iii) the value of $\mu$.
4. (a) Two particles of masses 7 kg and 3 kg are connected by a taut, light, inelastic string which passes over a smooth light pulley. The system is released from rest.

Find (i) the common acceleration of the particles.
(ii) the tension in the string.

(b) A rough plane is inclined at $30^{\circ}$ to the horizontal and has a smooth light pulley attached to its uppermost point.
A taut, light, inelastic string passes over the pulley and has masses of 3 kg and 2 kg attached to its end points. The coefficient of friction
 between the 2 kg mass
and the plane is $\frac{1}{\sqrt{3}}$.
The 3 kg mass hangs vertically.
The system is released from rest.
The 3 kg mass moves vertically downwards.
(i) Show on separate diagrams all the forces acting on each mass.
(ii) Find the common acceleration.
(iii) Find the tension in the string.
4. (a) Two particles of masses 14 kg and 21 kg are connected by a light, taut, inextensible string passing over a smooth light pulley at the edge of a rough horizontal table.
The coefficient of friction between the 14 kg mass and the table is $\frac{1}{2}$.

The system is released from rest.
(i) Show on separate diagrams the forces acting on each particle.
(ii) Find the common acceleration of the particles.
(b) A light inelastic string passes over a smooth light pulley. A mass of $x \mathrm{~kg}$ is attached to one end of the string and a mass of 2 kg is attached to the other end.

When the system is released from rest the 2 kg mass falls 3 metres in $\sqrt{6}$ seconds.

Find
(i) the common acceleration

(ii) the tension in the string
(iii) the value of $x$.
4. A particle of mass $M \mathrm{~kg}$ is placed on a rough plane inclined at $30^{\circ}$ to the horizontal.

This particle is connected by a light inextensible string passing over a smooth light pulley at the top of the plane to a particle of mass 20 kg , hanging freely under gravity.

The coefficient of friction between the $M \mathrm{~kg}$
 mass and the plane is $\frac{2}{5 \sqrt{3}}$.
The system is released from rest.
The 20 kg mass moves vertically upwards a distance of 16 m in 4 s .
(i) Show on separate diagrams all the forces acting on each particle.
(ii) Show that the constant acceleration of the particles is $2 \mathrm{~m} / \mathrm{s}^{2}$.
(iii) Find the tension in the string.
(iv) Find the value of $M$.
4. (a) Two particles, of masses 8 kg and 12 kg , are connected by a light, taut, inextensible string passing over a smooth light pulley at the edge of a smooth horizontal table.

The 12 kg mass hangs freely under gravity.
The particles are released from rest.
The 12 kg mass moves vertically downwards.

(i) Show on separate diagrams all the forces acting on each particle.
(ii) Find the acceleration of the 12 kg mass.
(iii) Find the tension in the string.
(b) A particle of mass 6 kg is placed on a rough plane inclined at an angle of $45^{\circ}$ to the horizontal.
The coefficient of friction between the particle and the plane is $\mu$. The particle is released from rest and takes
4 seconds to move a distance of $10 \sqrt{2}$ metres
 down the plane.
(i) Show on a diagram all the forces acting on the particle.
(ii) Show that the acceleration of the particle is $\frac{5 \sqrt{2}}{4} \mathrm{~m} / \mathrm{s}^{2}$.
(iii) Find the value of $\mu$.
4. (a) Two particles, of masses 10 kg and $M \mathrm{~kg}$, are connected by a light, taut, inextensible string passing over a smooth light pulley at the edge of a rough horizontal table.

The coefficient of friction between the 10 kg mass and the table is $\frac{1}{2}$.


The $M \mathrm{~kg}$ mass hangs freely under gravity.
The particles are released from rest.
The $M \mathrm{~kg}$ mass moves vertically downwards with an acceleration of $4 \mathrm{~m} / \mathrm{s}^{2}$.
(i) Show on separate diagrams all the forces acting on each particle.
(ii) Find the tension in the string.
(iii) Find the value of $M$.
(b) Calculate the initial speed that a stone must be given to make it skim horizontally across ice so that it comes to rest after skimming 40 m .
The coefficient of friction between the stone and the ice is $\frac{1}{8}$.
4. Particles, of masses 2 kg and 3 kg , resting on a rough horizontal table, are connected by a light taut inextensible string.
The coefficient of friction between the 2 kg mass and the table is $\frac{1}{8}$ and
between the 3 kg mass and the table is $\frac{1}{4}$.
The 3 kg mass is connected by a second light inextensible string passing over a smooth light pulley at the edge of the table to a particle of mass 5 kg . The 5 kg mass hangs freely under gravity.
The particles are released from rest.
The 5 kg mass moves vertically downwards.
(i) Show on separate diagrams all the forces acting on each particle.
(ii) Write down the equation of motion for each particle.
(iii) Find the common acceleration of the particles and the tension in each string.
4. (a) Two particles, of masses 18 kg and 9 kg respectively, are connected by a light inextenstible string passing over a smooth light pulley at the edge of a rough horizontal table. The coefficient of friction between the 18 kg mass and the table is $\mu$.

The 9 kg mass hangs freely under gravity.
The particles are released from rest.
The 9 kg mass moves vertically downwards with
 an acceleration of $\frac{5}{9} \mathrm{~m} / \mathrm{s}^{2}$.
(i) Show on separate diagrams all the forces acting on each particle.
(ii) Find the value of the tension in the string.
(iii) Find the value of $\mu$, giving your answer as a fraction.
(b) A particle of mass 20 kg is placed on a rough plane inclined at an angle A to the horizontal where $\tan \mathrm{A}=\frac{3}{4}$. This particle is connected by means of a light inextensible string passing over a smooth light pulley at the top of the plane to a particle of mass $m \mathrm{~kg}$, hanging freely under gravity.
 The coefficient of friction between the 20 kg mass and the plane is $\frac{1}{4}$.

The system is released from rest. The 20 kg mass moves up the plane. The value of the tension in the string is 200 Newtons.
(i) Find the common acceleration of the particles.
(ii) Show that $m=25$.
4. A particle of mass 20 kg is placed on a rough plane inclined at an angle $30^{\circ}$ to the horizontal.
The particle is on the point of moving down the plane.
(i) Show on a diagram all the forces acting
 on the particle.
(ii) Find the value of $\mu$, the coefficient of friction between the particle and the plane.
Give your answer in the form $\frac{1}{\sqrt{p}}, p>0$.

A smooth light pulley is now attached to the top of this plane. A particle of mass $m \mathrm{~kg}$, hanging freely under gravity, is now connected to the particle of mass 20 kg by means of a light inextensible string passing over this smooth pulley at the top of the plane.

The particles are released from rest.


The 20 kg particle moves with an acceleration of $2 \mathrm{~m} / \mathrm{s}^{2}$ up the plane.

Find the value of $m$ and the value of the tension in the string.

## Answers

## 2011(a)

(ii) $15 / 4 \mathrm{~ms}^{-2}$
(iii) $25 \mathrm{~g} / 8 \mathrm{~N}$ or $125 / 4 \mathrm{~N}$

## 2011(b)

## (ii) 2 ms

(iii) 24 N

## 2010 (a)

(i) $\mathrm{a}=1.67 \mathrm{~m} \mathrm{~s}^{-2}$
(ii) $\mathrm{T}=58.3 \mathrm{~N}$

## 2010 (b)

(i) $\mathrm{a}=5 / 9 \mathrm{~m} \mathrm{~s}^{-2}$
(ii) $\mathrm{T}=44.4 \mathrm{~N}$

2009 (a)
(i) $\mathrm{a}=4 \mathrm{~m} \mathrm{~s}^{-2}$
(ii) $\mathrm{T}=12 \mathrm{~N}$

2009 (b)
(i) $\mathrm{a}=5 \mathrm{~m} \mathrm{~s}^{-2}$

## 2008 (a)

(i) $\mathrm{a}=2.86 \mathrm{~m} \mathrm{~s}^{-2}$
(ii) $\mathrm{T}=64.29 \mathrm{~N}$

2007 (a)
(i) $\mathrm{a}=4 \mathrm{~m} \mathrm{~s}^{-2}$
(ii) $\mathrm{T}=42 \mathrm{~N}$

2007 (b)
(i)
(ii) $\mathrm{a}=2 \mathrm{~m} \mathrm{~s}^{-2}$
(iii) $\mathrm{T}=24 \mathrm{~N}$

2006 (a)
(i)
(ii) $\mathrm{a}=4 \mathrm{~m} \mathrm{~s}^{-2}$

## 2006 (b)

(i) $\mathrm{a}=1 \mathrm{~ms}^{-2}$
(ii) $\mathrm{T}=18 \mathrm{~N}$
(iii) $\mathrm{x}=18 / 11 \mathrm{~kg}$

2005
(i)
(ii)
(iii) $\mathrm{T}=240 \mathrm{~N}$
(iv) $\mathrm{M}=240 \mathrm{~kg}$

2003 (a)
(i)
(ii) $\mathrm{T}=90 \mathrm{~N}$
(iii) $\mathrm{M}=15 \mathrm{~kg}$

2003 (b)
$\mathrm{u}=10 \mathrm{~m} \mathrm{~s}^{-1}$

## 2002

(i)
(ii) $\mathrm{T}_{1}-1 / 8(2 g)=2 \mathrm{a}$
$\mathrm{T}_{2}-\mathrm{T}_{1}-1 / 4(3 g)=3 \mathrm{a}$
$5 g-\mathrm{T}_{2}=5 \mathrm{a}$
(iii) $\mathrm{a}=4 \mathrm{~m} \mathrm{~s}^{-2}$
$\mathrm{T}_{1}=10.5 \mathrm{~N}, \mathrm{~T}_{2}=30 \mathrm{~N}$

## 2001 (a)

(i)
(ii) $\mathrm{T}=85 \mathrm{~N}$
(iii) $\mu=5 / 12$

2001 (b)
(i) $\mathrm{a}=2 \mathrm{~m} \mathrm{~s}^{-1}$
(ii)

## 2004 (a)

(i)
(ii) $\mathrm{a}=6 \mathrm{~m} \mathrm{~s}^{-2}$
(iii) $\mathrm{T}=48 \mathrm{~N}$

## 2000 (a)

(i)

## 2004 (b)

(i)
(ii)
(iii) $\mu=0.75$

