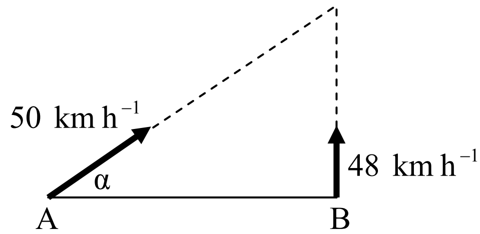
Relative Velocity

2011

2. Ship A is 126 km due west of ship B.
A is moving at a constant speed of 50 km h^{-1}
in the direction east α north where $\tan \alpha = \frac{24}{7}$.

B is moving due north
at a constant speed of 48 km h^{-1}.



Find **(i)** the velocity of A in terms of $\vec{i}$ and $\vec{j}$

(ii) the velocity of B in terms of $\vec{i}$ and $\vec{j}$

(iii) the velocity of A relative to B in terms of $\vec{i}$ and $\vec{j}$.

Ship A intercepts ship B after t hours.

Find **(iv)** the value of t

(v) the distance each ship travels in this time t.

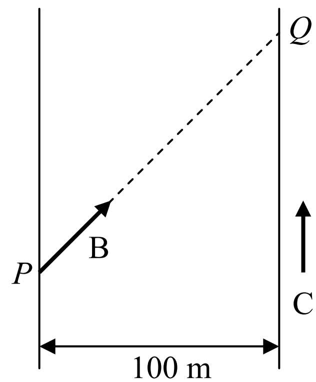
2010

2. A river is 100 metres wide and has parallel banks.

Boat B departs from point P on its western bank and lands at point Q on its eastern bank.

The actual velocity of the boat
is $5\vec{i} + 12\vec{j}$ ms^{-1}.

Cyclist C travels due north at a constant speed of 3 ms^{-1} along the eastern bank of the river.



Find **(i)** the velocity of C in terms of $\vec{i}$ and $\vec{j}$

(ii) the velocity of B relative to C in terms of $\vec{i}$ and $\vec{j}$

(iii) the magnitude and direction of the velocity of B relative to C

(iv) the time it takes B to cross the river

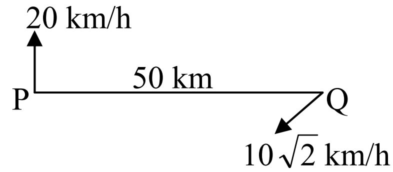
(v) $|PQ|$, the distance from P to Q.

2009

2. A ship P is moving north at a constant speed of 20 km/h.

Another ship Q is moving south-west at a constant speed of $10\sqrt{2}$ km/h.

At a certain instant, P is positioned 50 km due west of Q.



Find **(i)** the velocity of P in terms of $\vec{i}$ and $\vec{j}$

(ii) the velocity of Q in terms of $\vec{i}$ and $\vec{j}$

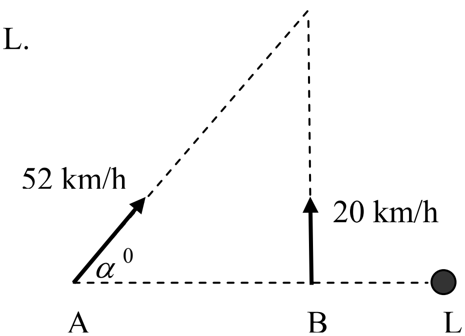
(iii) the velocity of P relative to Q in terms of $\vec{i}$ and $\vec{j}$

(iv) the shortest distance between P and Q in the subsequent motion.

2008

2. Ship A is 432 km due west of ship B.
Ship B is 135 km due west of lighthouse L.
A is travelling at a constant speed of 52 km/h in the direction east α^0 north, where $\tan\alpha = \frac{5}{12}$.

B is travelling due north at a constant speed of 20 km/h.



Find **(i)** the velocity of A in terms of $\vec{i}$ and $\vec{j}$

(ii) the velocity of B in terms of $\vec{i}$ and $\vec{j}$

(iii) the velocity of A relative to B in terms of $\vec{i}$ and $\vec{j}$.

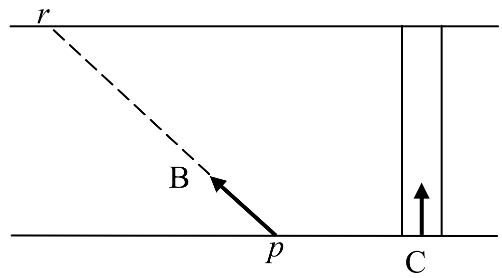
Ship A intercepts ship B after *t* hours.

(iv) Find the value of *t*.

(v) Find the distance from lighthouse L to the meeting point.

2007

2. A river is 72 metres wide and has parallel banks.
A boat B departs from point p on the southern bank and lands at point r on the northern bank.



The actual velocity of B is $-4\vec{i}+3\vec{j}$ m/s.
Cyclist C travels due north at a constant speed of 4 m/s across a straight level bridge which spans the river.

Find

(i) the velocity of C in terms of $\vec{i}$ and $\vec{j}$

(ii) the velocity of B relative to C in terms of $\vec{i}$ and $\vec{j}$

(iii) the magnitude and direction of the velocity of B relative to C

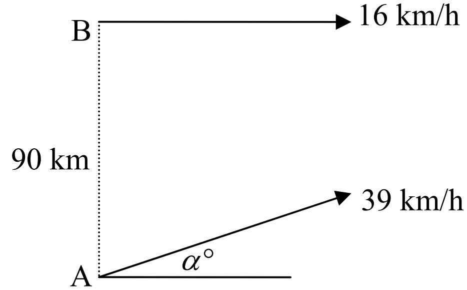
(iv) the time it takes C to cross the river

(v) how much longer it will take B to cross the river.

2006

2. Ship A is travelling east $\alpha°$ north with a constant speed of 39 km/h, where $\tan\alpha = \dfrac{5}{12}$.
Ship B is travelling due east with a constant speed of 16 km/h.

At 2 pm ship B is positioned 90 km due north of ship A.



(i) Express the velocity of ship A and the velocity of ship B in terms of $\vec{i}$ and $\vec{j}$.

(ii) Find the velocity of ship A relative to ship B in terms of $\vec{i}$ and $\vec{j}$.

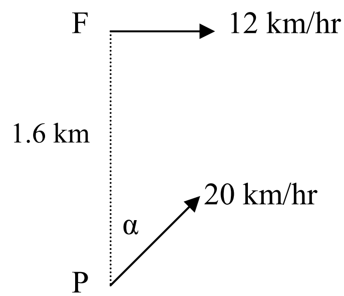
(iii) Find the shortest distance between the ships.

2005

2. **(a)** Two athletes A and B are running due east in a race.
At a certain instant athlete A is x metres from the finishing line and is running with a constant speed of 8 m/s. At this instant athlete B is 6 metres behind A and is running with a constant speed of 10 m/s.
B catches up with A at the finishing line, so that the race ends in a dead heat.

(i) Find the velocity of B relative to A.
(ii) Find the value of x.

(b) A ferry F is travelling due east with a constant speed of 12 km/hr.
A boat P is travelling in the direction α degrees east of north with a constant speed of 20 km/hr.
At noon P is 1.6 km due south of F and t minutes later P intercepts F.



(i) Find the velocity of P relative to F, in terms of $\vec{i}$, $\vec{j}$ and α.
(ii) Find the value of α, correct to the nearest degree.
(iii) Find the value of t.

2004

2. **(a)** Ship A is travelling due north with a constant speed of 15 km/hr.
Ship B is travelling north-west with a constant speed of $15\sqrt{2}$ km/hr.

(i) Write down the velocity of ship A and the velocity of ship B, in terms of $\vec{i}$ and $\vec{j}$.
(ii) Find the velocity of ship A relative to ship B.
(iii) If ship A is 5.5 km due west of ship B at noon, at what time will ship A intercept ship B?

(b) Car P and car Q are travelling eastwards on a straight level road.
P has a constant speed of 20 m/s and Q has a constant speed of 10 m/s.

(i) Find the velocity of P relative to Q.
(ii) At a certain instant car P is 100 m behind car Q.
Find the distance between the two cars 3.5 seconds later.

2003

2. The velocity of ship A is $3\vec{i} - 4\vec{j}$ m/s and the velocity of ship B is $-2\vec{i} + 8\vec{j}$ m/s.

(i) Find the velocity of ship A relative to ship B in terms of $\vec{i}$ and $\vec{j}$.

(ii) Find the magnitude and direction of the velocity of ship A relative to ship B, giving the direction to the nearest degree.

At a certain instant, ship B is 26 km due east of ship A.

(iii) Show, on a diagram, the positions of ship A and ship B at this instant and show, also, the direction in which ship A is travelling relative to ship B.

(iv) Calculate the shortest distance between the ships, to the nearest km.

2002

2. Ship A is travelling due west with a constant speed of 10 km/hr.
Ship B is travelling at a constant velocity.
At 1200 hours, the radar screen of ship A shows the position of ship B relative to ship A as $-2\vec{i} - 20\vec{j}$ kilometres.
At 1400 hours, two hours later, the position of ship B relative to ship A is $8\vec{i} + 4\vec{j}$ kilometres.

(i) Write down the velocity of ship A in terms of $\vec{i}$ and $\vec{j}$.

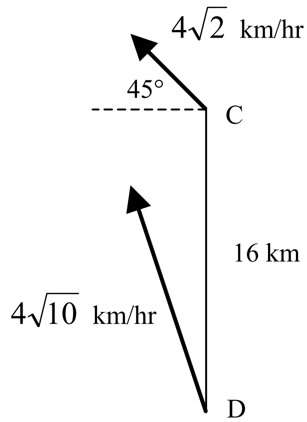
(ii) Show that the change in the position of ship B relative to ship A between 1200 hours and 1400 hours is $10\vec{i} + 24\vec{j}$ kilometres.

(iii) Find the velocity of ship B relative to ship A.

(iv) Find the speed and direction of ship B.
Give the direction to the nearest degree.

2001

2. At a certain instant ship D is 16 km due south of ship C.
Ship C is travelling with a speed of $4\sqrt{2}$ km/hr in a north-westerly direction.
Ship D is travelling with a speed of $4\sqrt{10}$ km/hr to intercept C.
Let the velocity of D be $x\vec{i} + y\vec{j}$ km/hr.



(i) Write down the velocity of C in terms of $\vec{i}$ and $\vec{j}$.
(ii) Find the value of x and the value of y.
(iii) How long does it take ship D to intercept ship C ?

2000

2. **(a)** Ship A is travelling with a speed of 15 km/hr in the direction due East.
Ship B is travelling with a speed of 20 km/hr in the direction due South.

Find the velocity of ship A relative to ship B.

(b) A river is 100 m wide and is flowing with a speed of 2 m/
banks. The speed of a swimmer in still water is 3 m/s.

(i) What is the shortest time it takes the swimmer to swim across the river?

(ii) What direction should the swimmer take so as to swim straight across to a point directly opposite?
How long will it then take the swimmer to cross to this point?

2010

(i) Vc = 0 i + 3 j

(ii) Vbc = 5 i + 9 j

(iii) $|Vbc|$ = 10.3, direction = E 60.9^0 N

(iv) Time = 20 s

(v) 260m

2009

(i) V_P = 0 i + 20 j

(ii) V_Q = - 10 i – 10 j

(iii) V_{PQ} = 10 i + 30 j

(iv) Shortest Distance = 47.43 km

2008

(i) V_A = 48 i + 20 j

(ii) V_B = 0 i + 20 j

(iii) V_{AB} = 48 i + 0 j

(iv) Time = 9 hours

(v) Distance = 225 km

2007

(i) V_c = 0 i + 4 j

(ii) V_{BC} = - 4 i - 1 j

(iii) Speed = 4.12 m s^{-1}, $\theta = 14.04^0$ with the bank

(iv) time = 18 s

(v) time = 24 s $\Rightarrow$ required time = 6 s

2006

(i) V_A = 36 i + 15 j

V_B = 16 i + 10 j

(ii) V_{AB} = 20 i + 15 j

(iii) Shortest Distance = 72 km

2005 (a)

(i) v_{BA} = 2i

(ii) x = 24 m

2005 (b)

(i) Vpf = (20 sinα - 12) i + 20 cos α j

(ii) $\alpha = 37^0$

(iii) t = 0.1 h or 6 minutes

2004 (a)

(i) V_A = 0 i + 15 j

V_B = -15 i + 15 j

(ii) V_{AB} = 15 i + 0 j

(iii) Time = 12:22

2004 (b)

(i) V_{PQ} = 10 i + 0 j

(ii) Distance = 65 m

2003

(i) V_{AB} = 5 i - 12 j

(ii) Magnitude = 13 m s^{-1}

Direction = 67^0 south of east

(iii)

(iv) Shortest distance = 24 km

2002

(i) V_A = -10 i + 0 j

(ii)

(iii) V_{AB} = 5 i + 12 j

(iv) Speed = 13 km/hr,

Direction = 67^0 N of W

2001

(i) v_C = -4i + 4j

(ii) y = 12

(iii) t = 2

2000 (a)

V_{AB} = 15i + 20j

2000 (b)

(i) time = 100/3 s

(ii) Direction: sin α = 2/3

time = $100/\sqrt{5}$